Wykorzystanie filtrów cyfrowych w układzie sterowania bezpośredniego napędu elektrycznego

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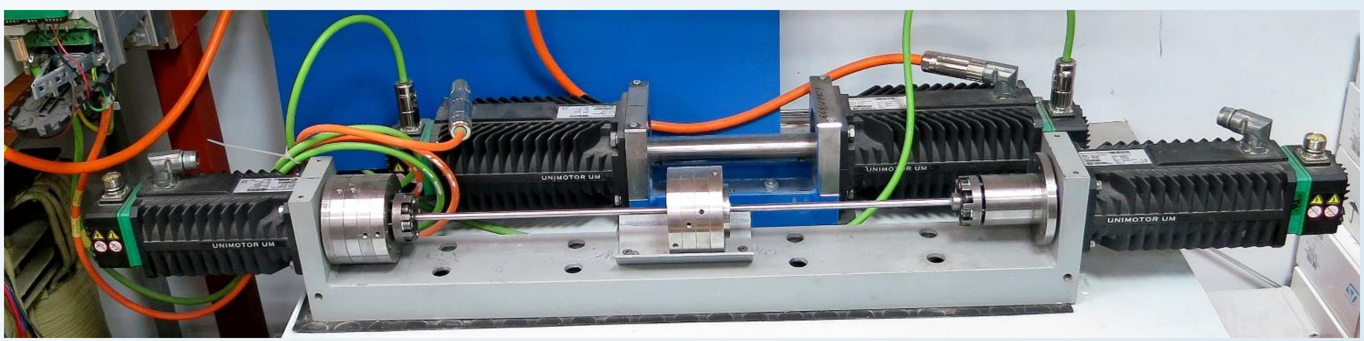
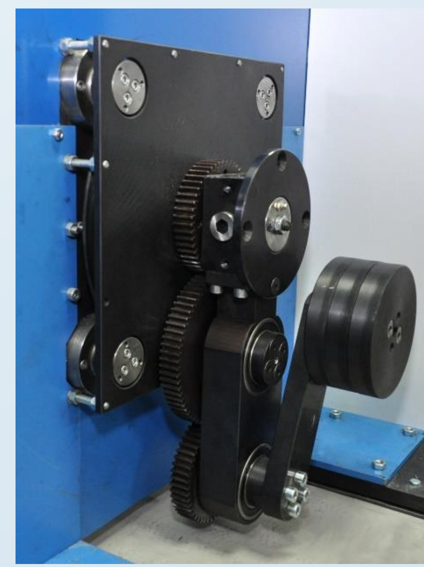
Plan of presentation

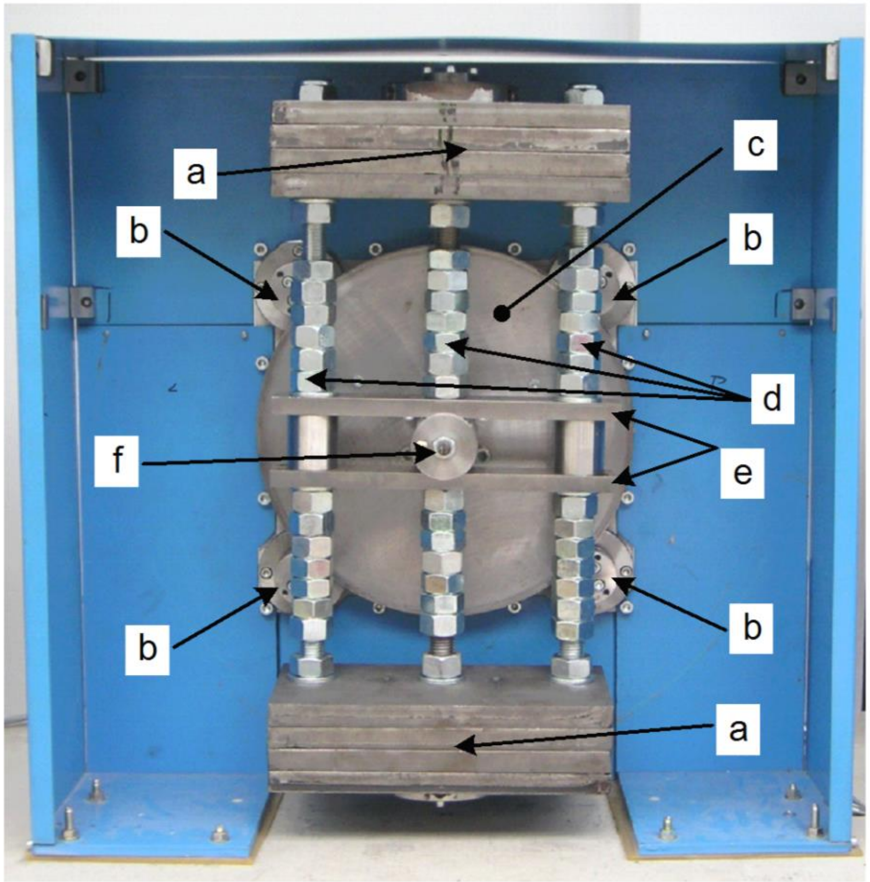
1. **Motivation**
2. Control system
3. Mechanical part of electric drive
4. Digital filter tuned for mechanical resonant frequency reduction
5. Results
6. Conclusions

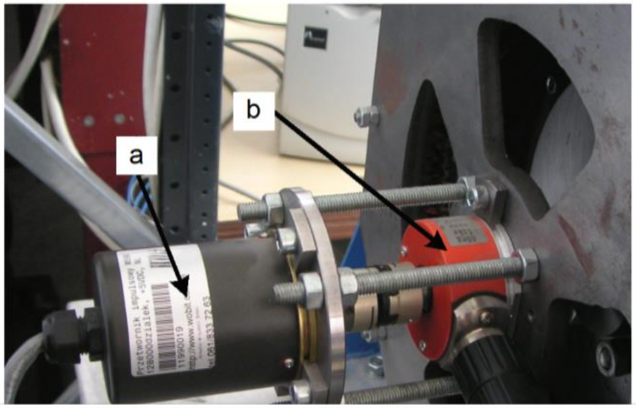
Multi mass real systems

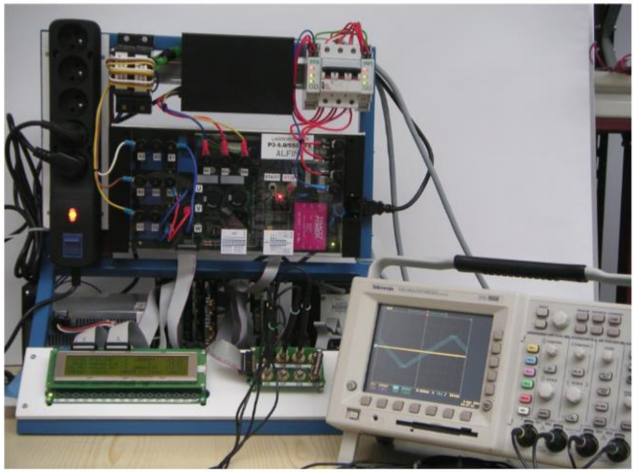




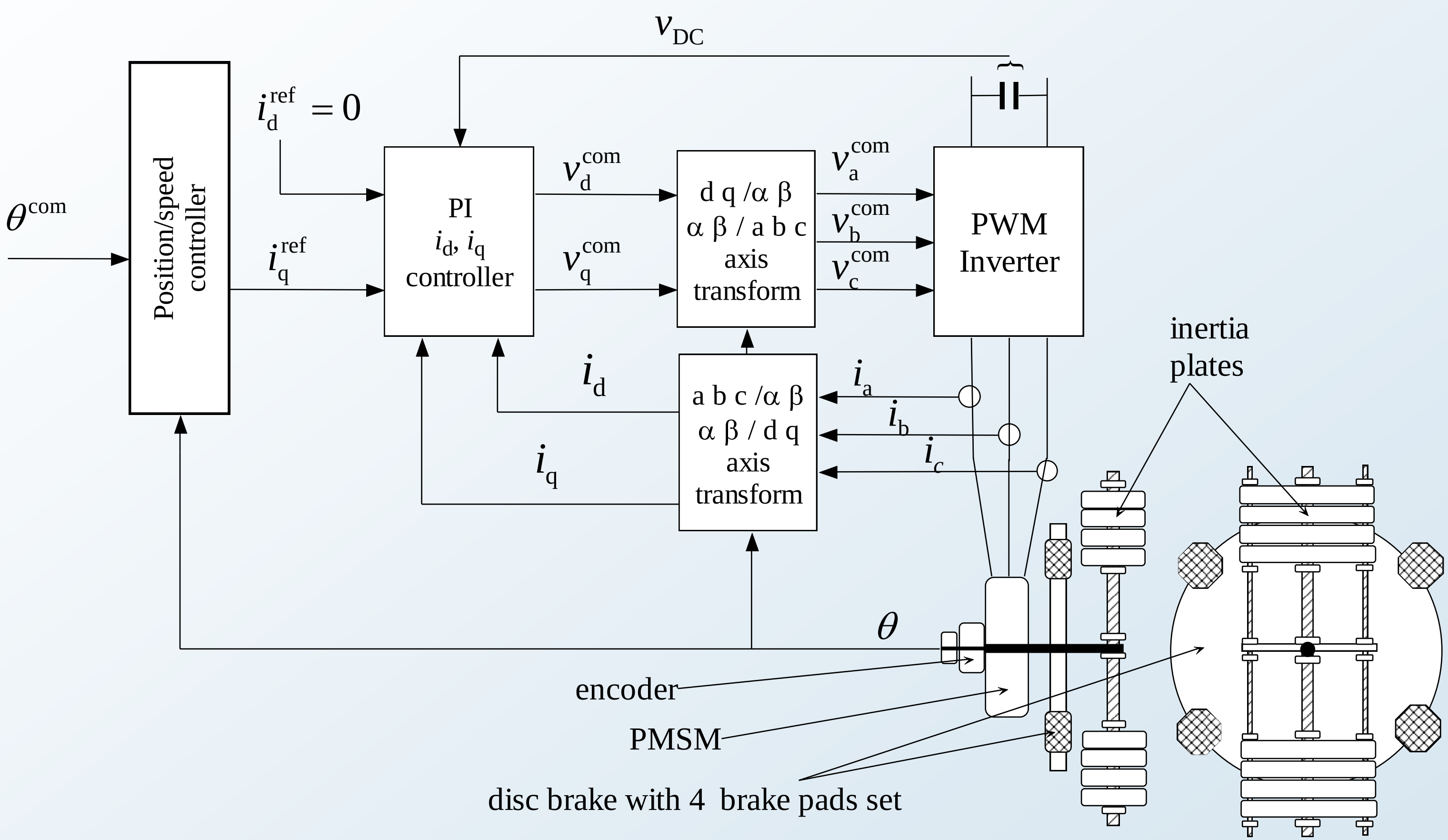
Experimental system







General system scheme



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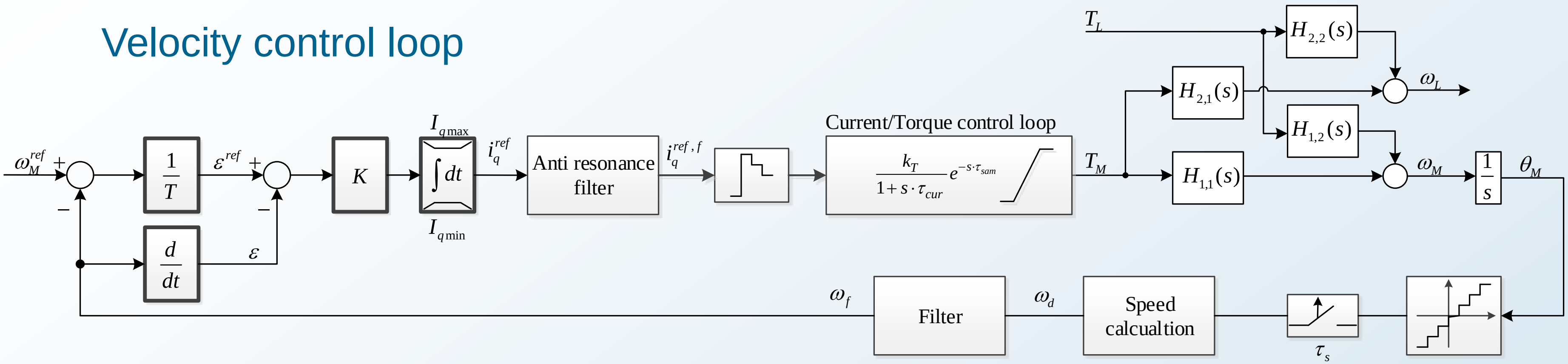
Discrete control system

Velocity control loop



Controller settings

$$K \approx 0.5 \frac{\pi}{2\tau_{dly}} \cdot \frac{J_{\min}}{k_{T\max}}$$

$$T \approx 2 \cdot \frac{J_{\max}}{K \cdot k_{T\min}}$$

$$J_{\max} = 1.1 \cdot J_{\Sigma}$$

$$J_{\min} = 0.9 \cdot J_{\Sigma}$$

Measurement error

$$\Delta\omega = \frac{2\pi}{N_p \cdot \tau_s} \approx 0{,}123 \text{ rad/s}$$

Total delay in control loop

$$\tau_{dly} = \tau_{sam} + \tau_{cur} + \tau_{filter} + \tau_{speed\ \mathrm{calculation}} + \tau_{anit-resonance\ filter}$$

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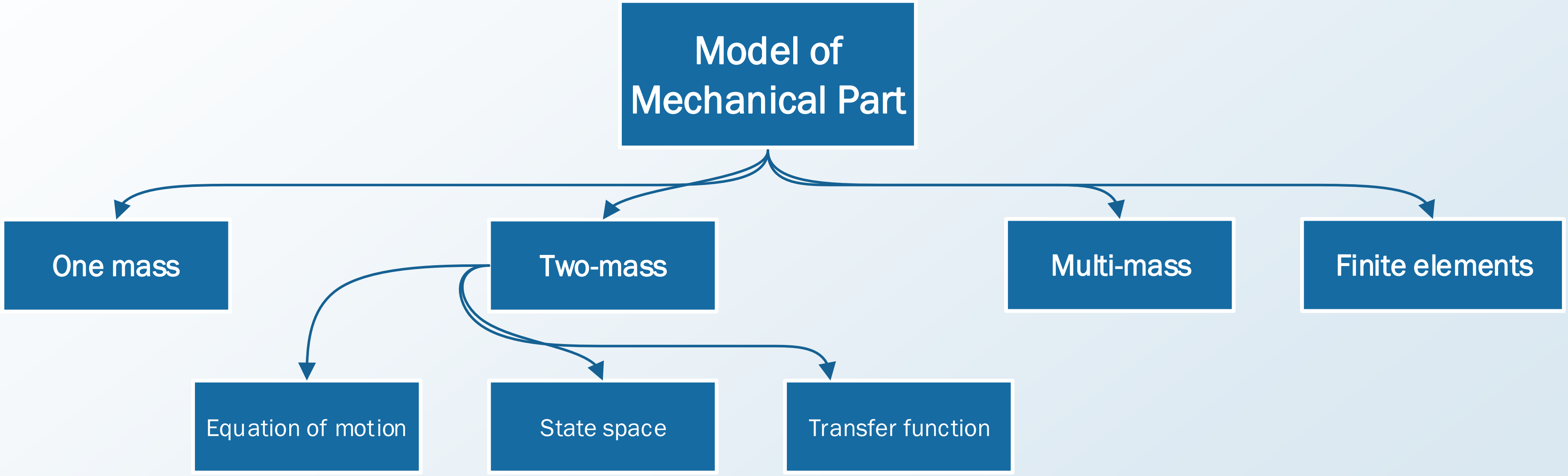
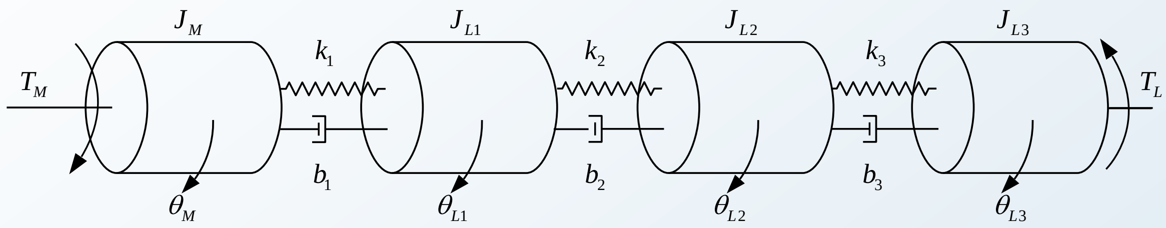
Models of mechanical part



Model of mechanical part – four-mass system

Scheme



Equations of motion

$$\begin{cases} J_M\ddot{\theta}_M + b_1(\dot{\theta}_M - \dot{\theta}_{L1}) + k_1(\theta_M - \theta_{L1}) = T_M \\ J_{L1}\ddot{\theta}_{L1} + b_2(\dot{\theta}_{L1} - \dot{\theta}_{L2}) + k_2(\theta_{L1} - \theta_{L2}) - b_1(\dot{\theta}_M - \dot{\theta}_{L1}) - k_1(\theta_M - \theta_{L1}) = 0 \\ J_{L2}\ddot{\theta}_{L2} + b_3(\dot{\theta}_{L2} - \dot{\theta}_{L3}) + k_3(\theta_{L2} - \theta_{L3}) - b_2(\dot{\theta}_{L1} - \dot{\theta}_{L2}) - k_2(\theta_{L1} - \theta_{L2}) = 0 \\ J_{L3}\ddot{\theta}_{L3} - b_3(\dot{\theta}_{L2} - \dot{\theta}_{L3}) - k_3(\theta_{L2} - \theta_{L3}) = -T_L \end{cases}$$

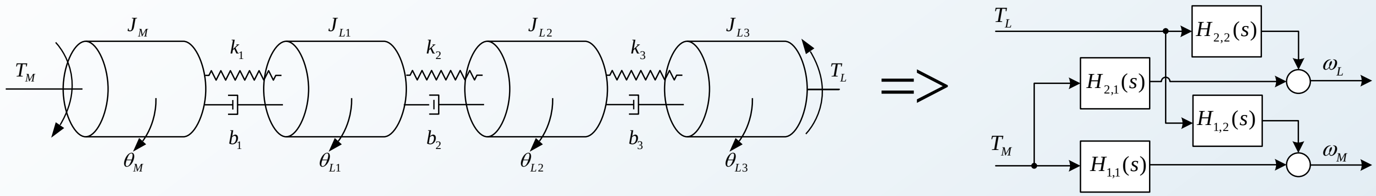
State space

$$\dot{\mathbf{x}} = \mathbf{A}_{7x7}\mathbf{x} + \mathbf{B}_{7x2}\begin{bmatrix} T_M \\ T_L \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \omega_M & \omega_{L1} & \omega_{L2} & \omega_{L2} & T_{s1} & T_{s2} & T_{s3} \end{bmatrix}^T$$

Model of mechanical part – four-mass system

Scheme



Transfer functions

$$\begin{bmatrix}\Omega_M(s)\\ \Omega_L(s)\end{bmatrix}=\begin{bmatrix}H_{1,1} & H_{1,2}\\ H_{2,1} & H_{2,2}\end{bmatrix}\begin{bmatrix}T_M(s)\\ T_L(s)\end{bmatrix}$$

$$H_{1,1}(s)=\frac{\omega_M(s)}{T_M(s)}=\frac{1}{J_\Sigma}\cdot\frac{1}{s}\cdot\prod_{i=1}^{L_R}G_{11,i}$$

$$H_{1,2}(s)=-H_{2,1}(s)$$

$$H_{2,1}(s)=\frac{\omega_{L3}(s)}{T_M(s)}=\frac{1}{J_\Sigma}\cdot\frac{1}{s}\cdot\prod_{i=1}^{L_R}G_{21,i}$$

$$H_{2,2}(s)=\frac{\omega_{L3}(s)}{T_L(s)}=\frac{1}{J_\Sigma}\cdot\frac{1}{s}\cdot\prod_{i=1}^{L_R}G_{22,i}$$

$$G_{poles,i}=\frac{\omega_{r,i}^2}{s^2+2\xi_{r,i}\omega_{r,i}s+\omega_{r,i}^2}$$

$$R_i=J_{i+1}/J_i$$

$$G_{11,i}=\frac{s^2+2\xi_{ar,i}\omega_{ar,i}s+\omega_{ar,i}^2}{\omega_{ar,i}^2}\cdot G_{poles,i}$$

$$G_{21,i}=\frac{2\xi_{ar,i}\omega_{ar,i}s+\omega_{ar,i}^2}{\omega_{ar,i}^2}\cdot G_{poles,i}$$

$$G_{22,i}=\frac{R_i^{-1}s^2+2\xi_{ar,i}\omega_{ar,i}s+\omega_{ar,i}^2}{\omega_{ar,i}^2}\cdot G_{poles,i}$$

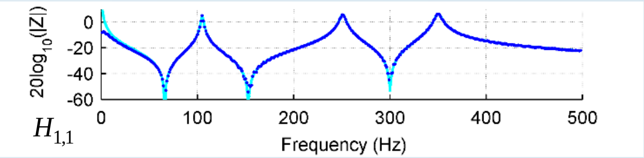
$\omega_{r,i}=2\cdot\pi\cdot f_{r,i}$ resonance frequency,

$\omega_{ar,i}=2\cdot\pi\cdot f_{ar,i}$ anti-resonance frequency,

$\xi_{r,i}$ $\xi_{ar,i}$ resonance and anti-resonance damping coefficients.

Tab. 1 Parameters of simulation model

J_Σ	i	$f_{ar,i}$	$\xi_{ar,i}$	$f_{r,i}$	$\xi_{r,i}$
(kg·m²)	-	(Hz)	*10⁻⁴	(Hz)	*10⁻⁴
0.753	1	66	10	105	90
	2	153	5	251	100
	3	300	10	350	90



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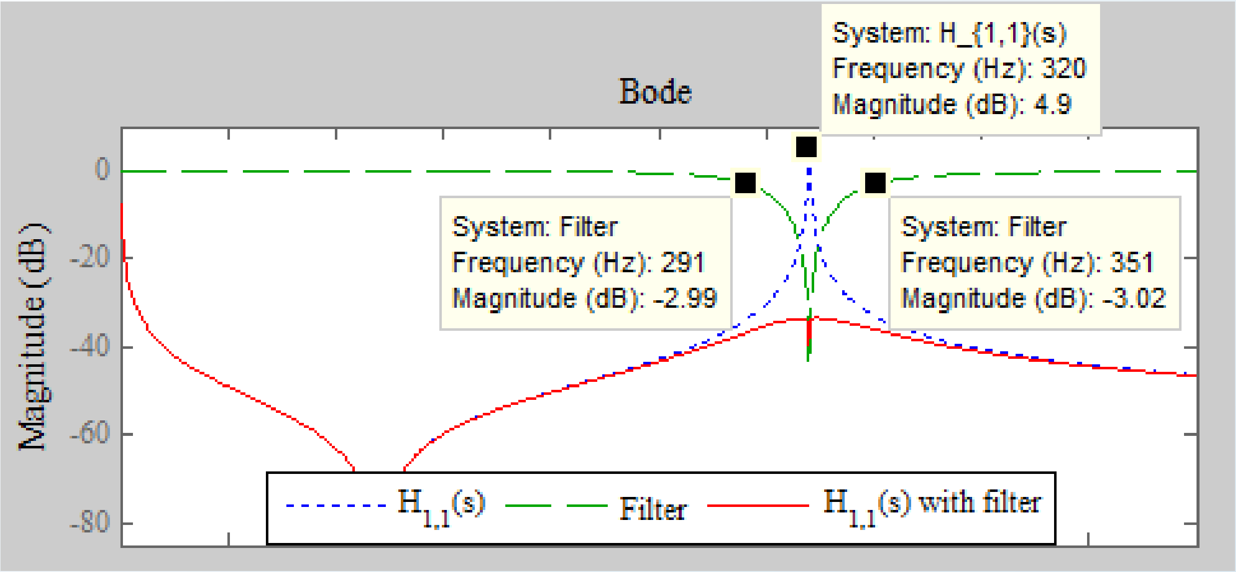
Notch filter

$$T_F(s) = \frac{s^2 + \omega_z^2}{s^2 + s\frac{\omega_z}{Q} + \omega_z^2}$$

$$\omega_z = 2\pi \cdot \mathrm{NTF}$$

$$Q = \mathrm{NTF}/f_{width}$$



NTF=320 Hz

f_{width} =60 Hz

Filter delay

$$H(j\omega) = P(\omega) + jQ(\omega)$$

$$\varphi(\omega) = \arg(H(j\omega)) = arctg\frac{Q(\omega)}{P(\omega)}$$

$$\tau_p(\omega) = -\frac{\varphi(\omega)}{\omega}$$ Phase delay

$$\tau_{gr}(\omega) = -\frac{d\varphi(\omega)}{d\omega}$$ Group delay

Notch filter

$$T_F(s) = \frac{s^2 + \omega_z^2}{s^2 + s\frac{\omega_z}{Q} + \omega_z^2}$$

$$\varphi(\omega) = arctg\left(\frac{\omega_z \omega}{Q(\omega^2 - \omega_z{}^2)}\right)$$

$$\tau_{gr}(\omega) = \frac{Q\omega_z\left(\omega^2 + \omega_z^2\right)}{Q^2\left(\omega^2 - \omega_z^2\right)^2 + \omega^2\omega_z^2}$$

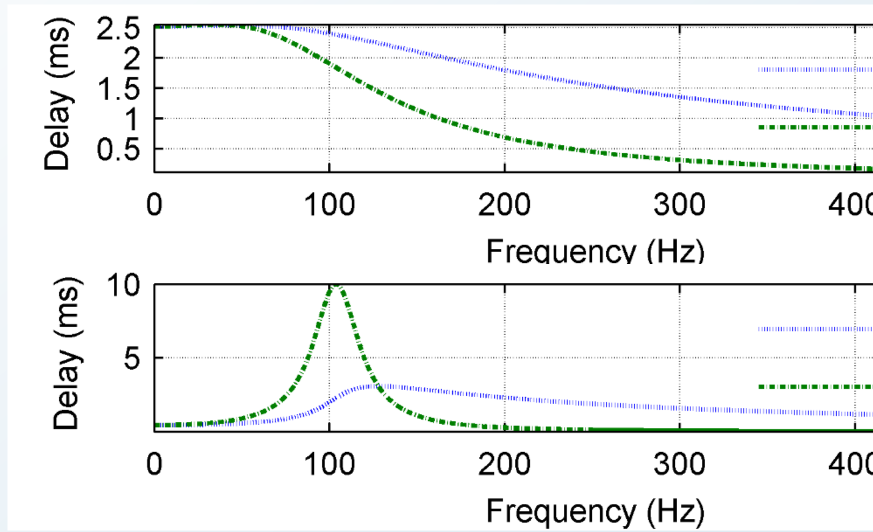
$$\omega_z = 2\pi \cdot \mathrm{NTF}$$

$$Q = \mathrm{NTF}/f_{width}$$

$$\tau_{gr}(0) = \frac{1}{Q\omega_z} = \frac{f_{width}}{2\pi \cdot \mathrm{NTF}^2}$$

Group and phase delay of notch filter

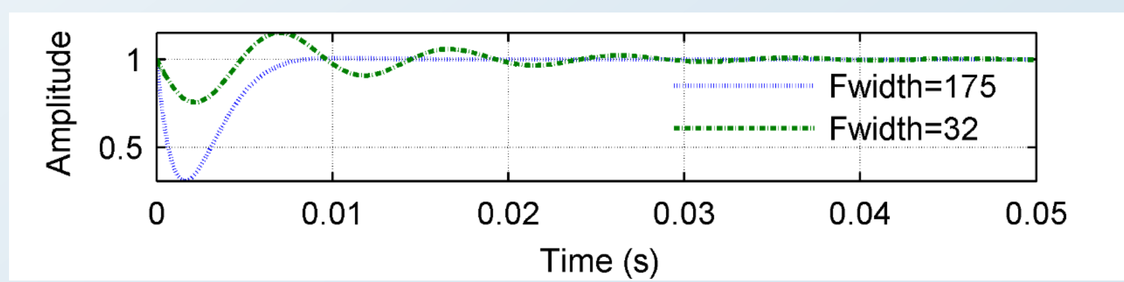
NTF=105 Hz



Fwidth=175
Q1=0.6

Fwidth=32
Q2=3.2813

Step response of notch filter



Notch filter

$$T_F(s) = \frac{s^2 + \omega_z^2}{s^2 + s\frac{\omega_z}{Q} + \omega_z^2}$$

$$\varphi(\omega) = arctg\left(\frac{\omega_z \omega}{Q(\omega^2 - \omega_z{}^2)}\right)$$

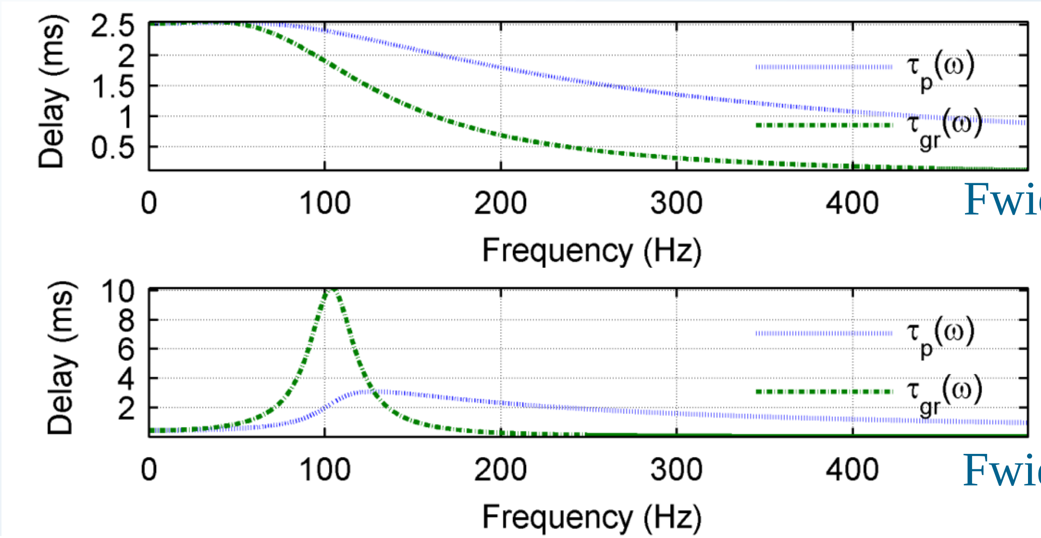
$$\tau_{gr}(\omega) = \frac{Q\omega_z\left(\omega^2 + \omega_z^2\right)}{Q^2\left(\omega^2 - \omega_z^2\right)^2 + \omega^2\omega_z^2}$$

$$\omega_z = 2\pi \cdot \mathrm{NTF}$$

$$Q = \mathrm{NTF}/f_{width}$$

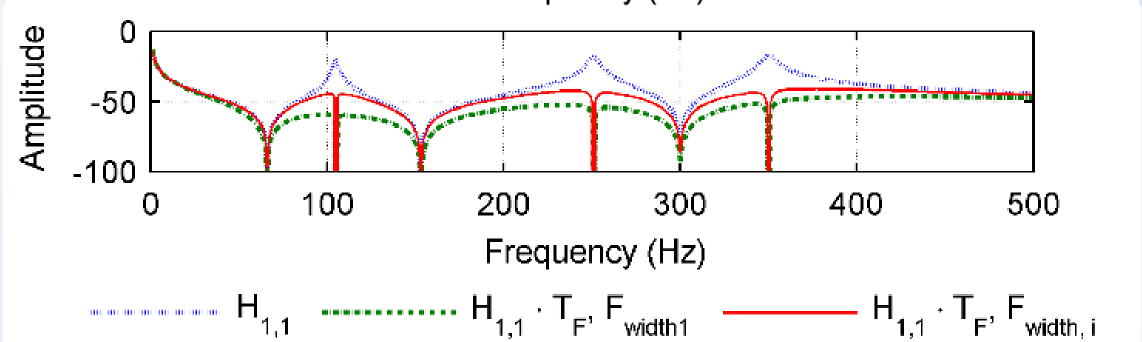
$$\tau_{gr}(0) = \frac{1}{Q\omega_z} = \frac{f_{width}}{2\pi \cdot \mathrm{NTF}^2}$$

Group and phase delay of notch filter



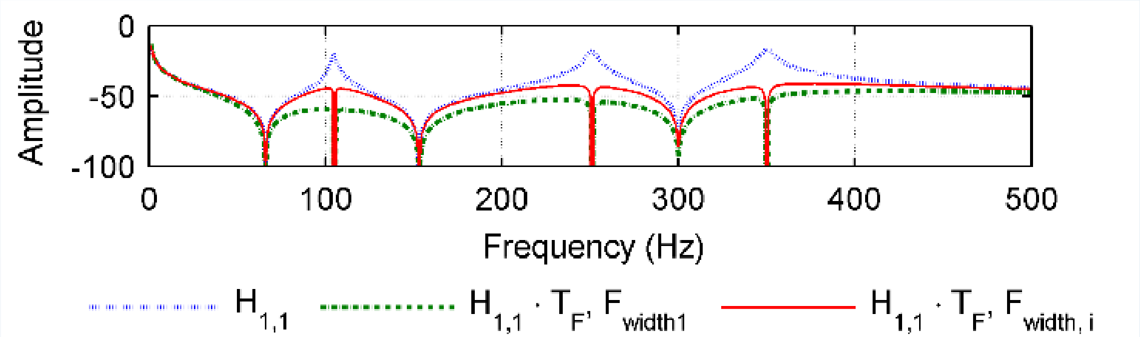
Fwidth=175

Fwidth=32



1)Fwidth1=175 2) Q=const=3.33 Fwidth, i={32, 75, 105}

Notch filter



$$T_F(s) = \frac{s^2 + \omega_z^2}{s^2 + s\frac{\omega_z}{Q} + \omega_z^2}$$

$$\varphi(\omega) = arctg\left(\frac{\omega_z \omega}{Q(\omega^2 - \omega_z{}^2)}\right)$$

$$\tau_{gr}(\omega) = \frac{Q\omega_z\left(\omega^2 + \omega_z^2\right)}{Q^2\left(\omega^2 - \omega_z^2\right)^2 + \omega^2\omega_z^2}$$

$$\omega_z = 2\pi \cdot \mathrm{NTF}$$

$$Q = \mathrm{NTF}/f_{width}$$

$$\tau_{gr}(0) = \frac{1}{Q\omega_z} = \frac{f_{width}}{2\pi \cdot \mathrm{NTF}^2}$$

1)Fwidth1=175

i	NTF	Fwidth	Q	Tgr(0) (ms)
1	105	175	0.6	2.526
2	251	175	1.4343	0.442
3	350	175	2	0.227

2) Q=const=3.33 Fwidth, i={32, 75, 105}

i	NTF	Fwidth	Q	Tgr(0) (ms)
1	105	31.532	3.33	0.455
2	251	75.375	3.33	0.190
3	350	105.11	3.33	0.136

Notch filter

$$T_F(s) = \frac{s^2 + \omega_z^2}{s^2 + s\frac{\omega_z}{Q} + \omega_z^2}$$

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$$\tau_{gr}(\omega) = \frac{Q\omega_z\left(\omega^2 + \omega_z^2\right)}{Q^2\left(\omega^2 - \omega_z^2\right)^2 + \omega^2\omega_z^2}$$

$$\omega_z = 2\pi \cdot \mathrm{NTF}$$

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i	NTF	Fwidth	Q	Tgr(0) , Σ 0.781 (ms)
1	105	31.532	3.33	0.455
2	251	75.375	3.33	0.190
3	350	105.11	3.33	0.136

3) Tgr(0)=const

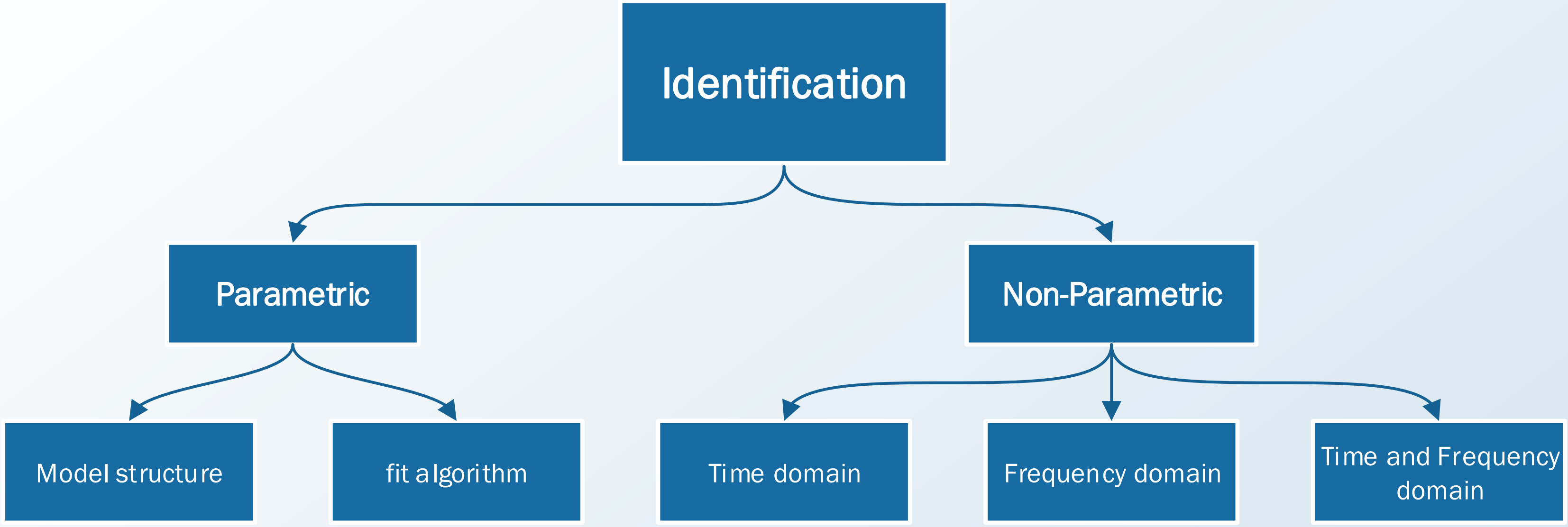
i	NTF	Fwidth	Q	Tgr(0), Σ 0.6 (ms)
1	105	13.854	7.5788	0.2
2	251	79.169	3.1704	0.2
3	350	153.94	2.2736	0.2

Digital filter tuned for mechanical resonant frequency reduction

Algorithm:

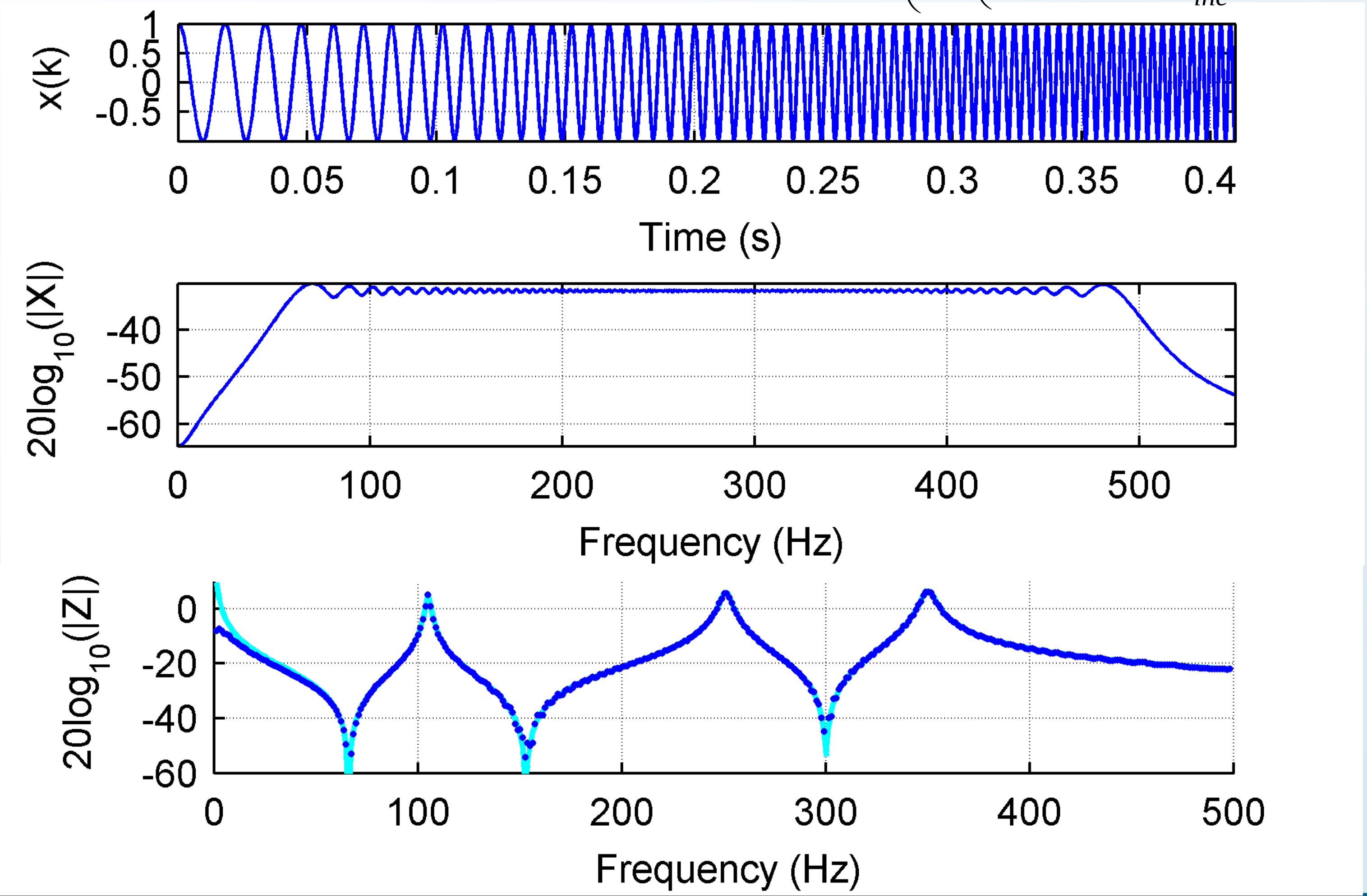
1. Identification of mechanical resonance frequencies.
2. Tune digital filter to attenuate mechanical resonance frequencies in command signal.
3. Calculate digital filter delay.
4. Tune speed controller depending on total system delay.

Identification of mechanical resonance

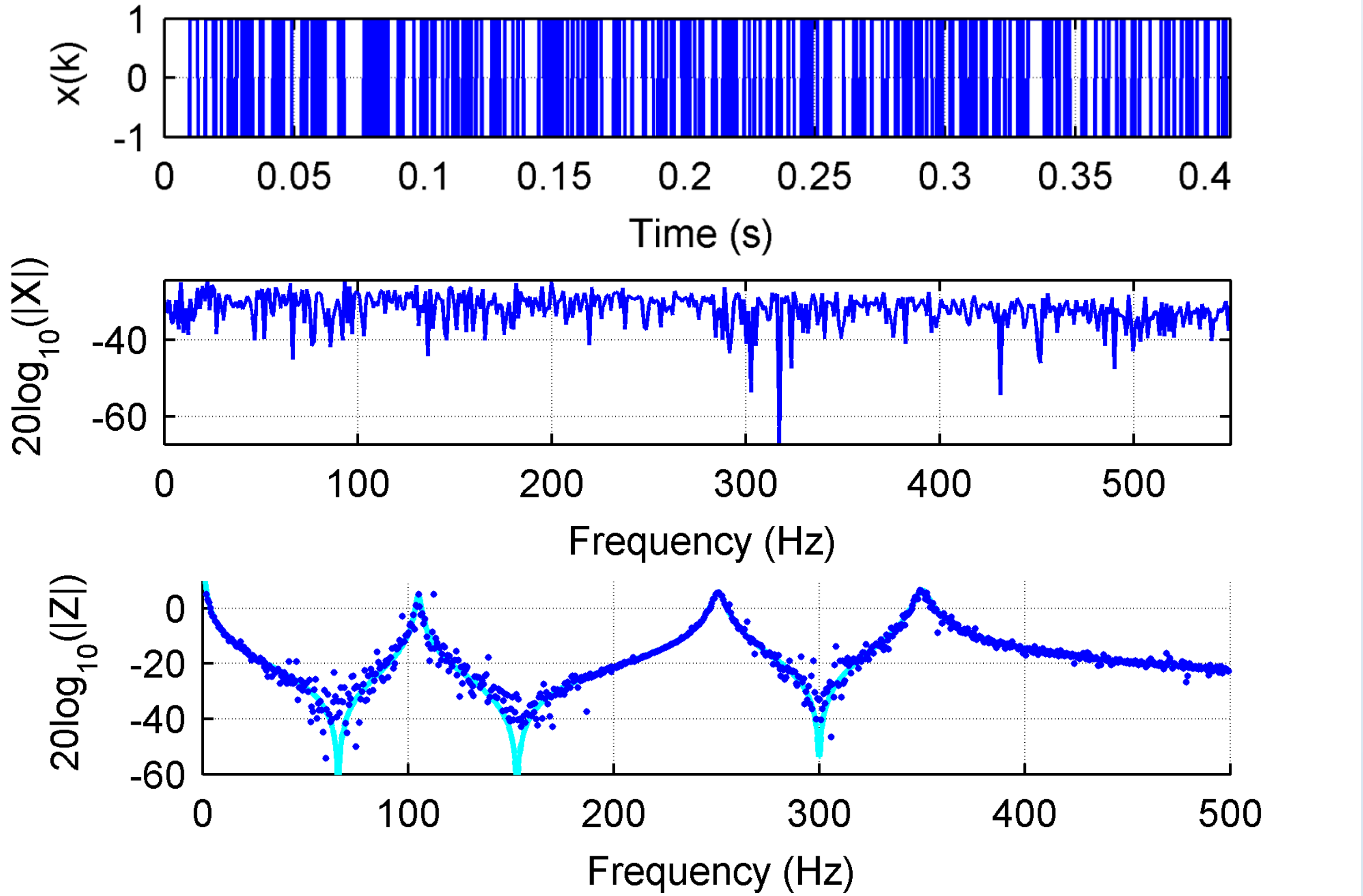


Non-parametric Linear chirp

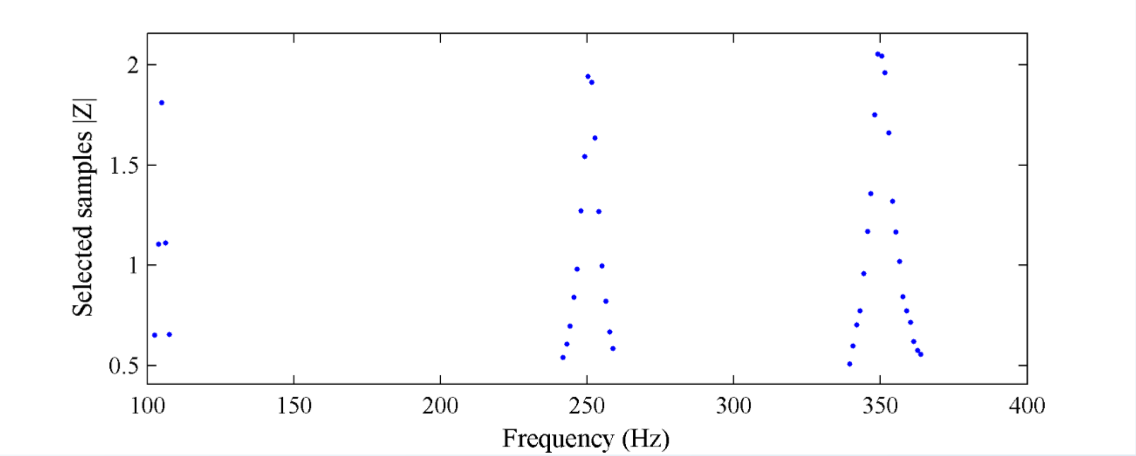
$$x(k) = A \cdot \cos\left(2\pi\left(f_{\min} + \frac{f_{\max} - f_{\min}}{2\tau_{inc}} k \cdot \tau_s\right) k \cdot \tau_s\right)$$



Non-parametric PRBS

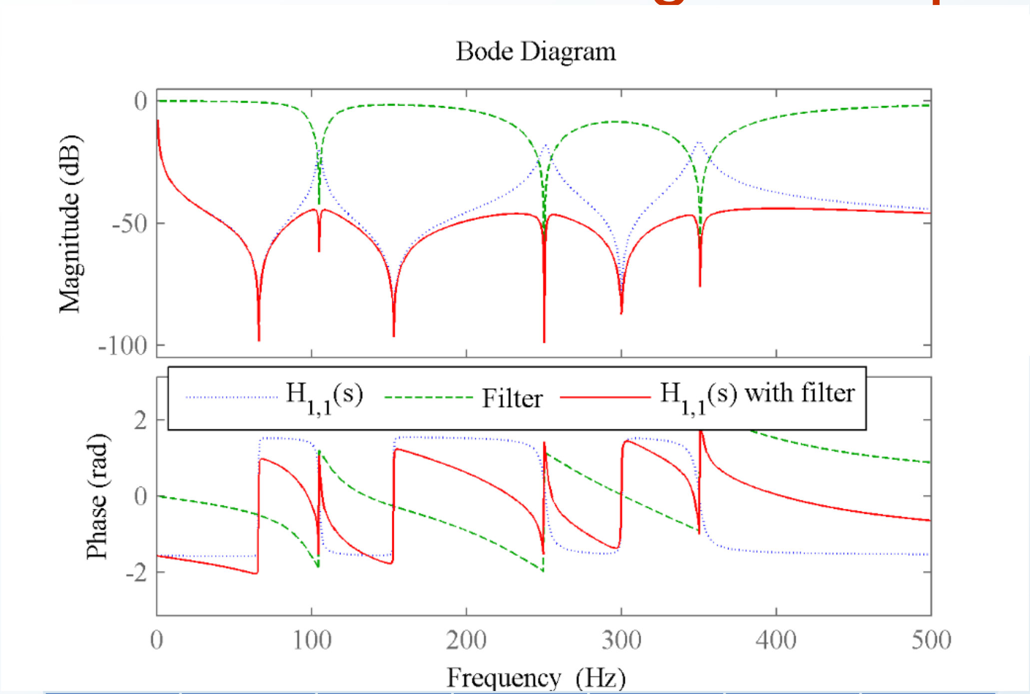


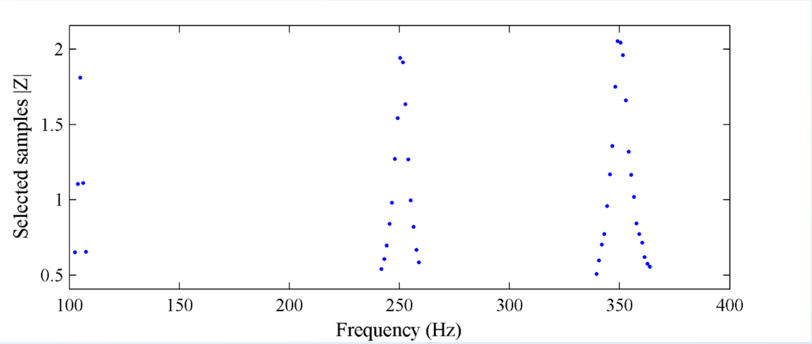
Non-parametric chirp – resonance finder



i	Fr,i	Fstart	Fstop	Fmax	Fwidth	Frez
1	*105*	102.5	107.4	105	4.9	**105**
2	*250*	241.7	258.8	250.2	17.1	**250.2**
3	*351*	339.4	363.8	349.1	24.1	**351.6**

Filters Auto Tuning - Non-parametric identyfication

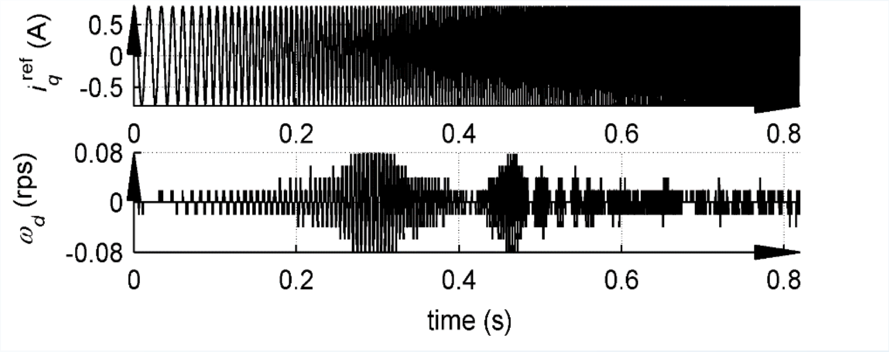




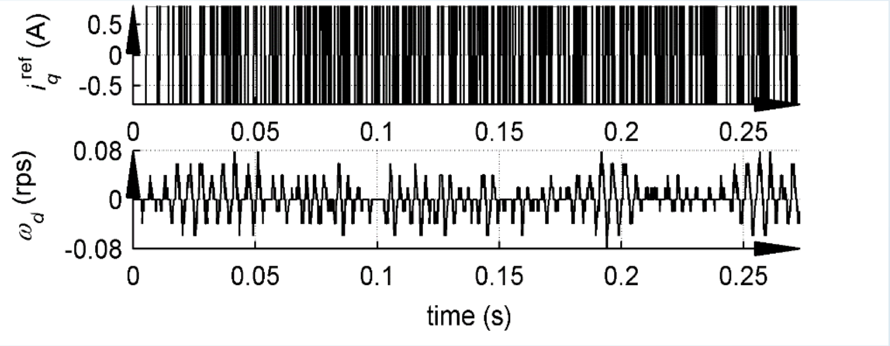
i	Fr,i	Fstart	Fstop	Fmax	Fwidt h	Frez
1	*105*	102.5	107.4	105	4.9	**105**
2	*250*	241.7	258.8	250.2	17.1	**250.2**
3	*351*	339.4	363.8	349.1	24.1	**351.6**

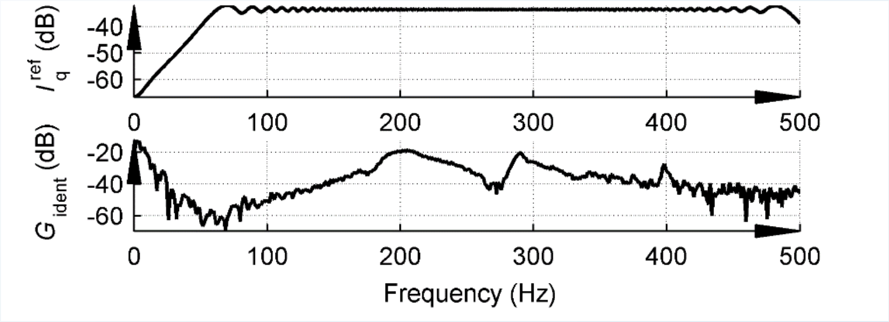
Experimental results

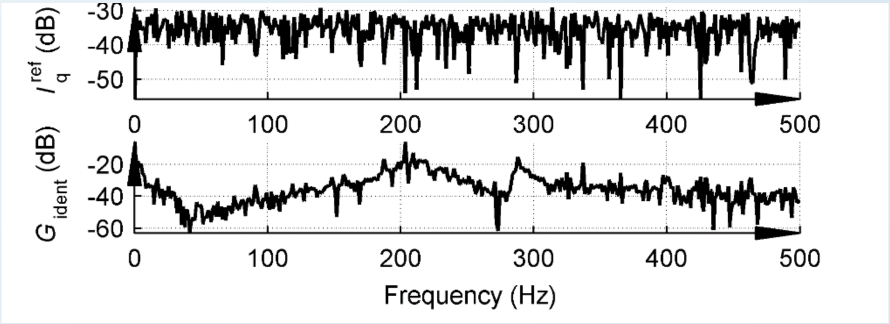
chirp



PRBS







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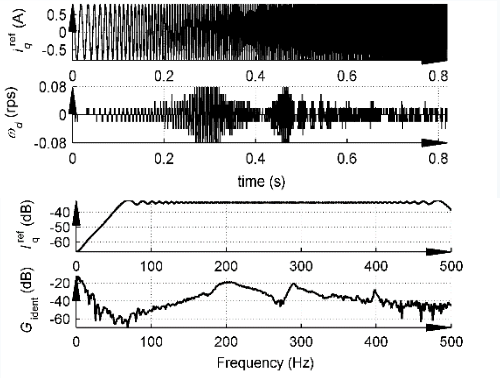
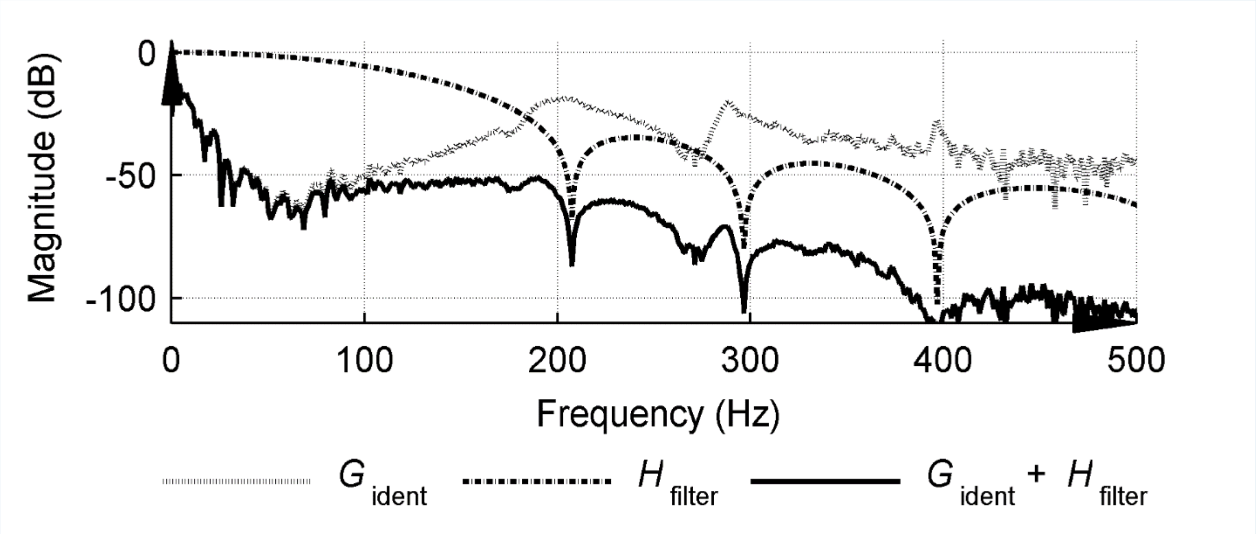
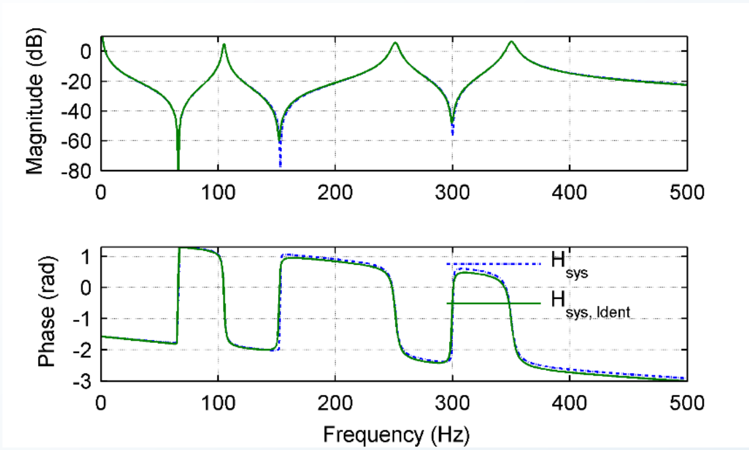
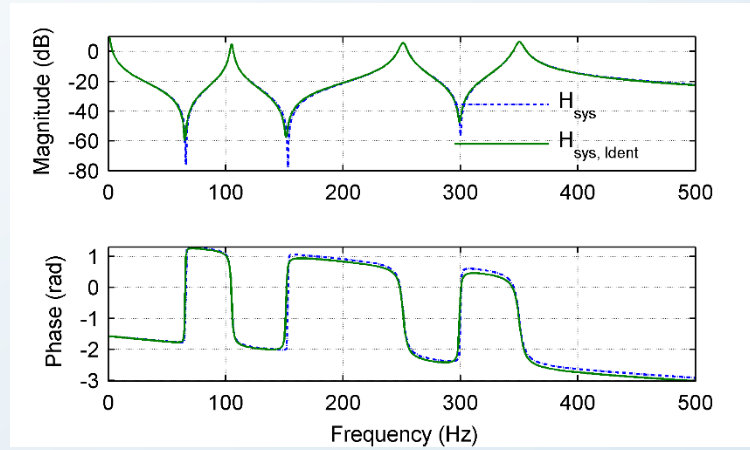




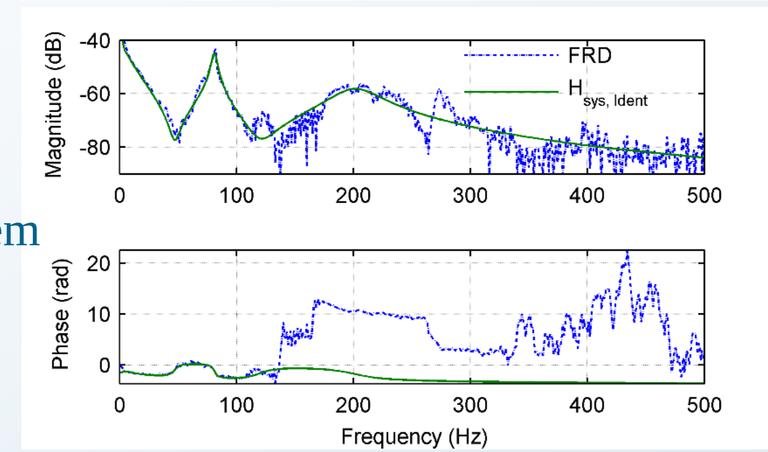
Parametric identyfication

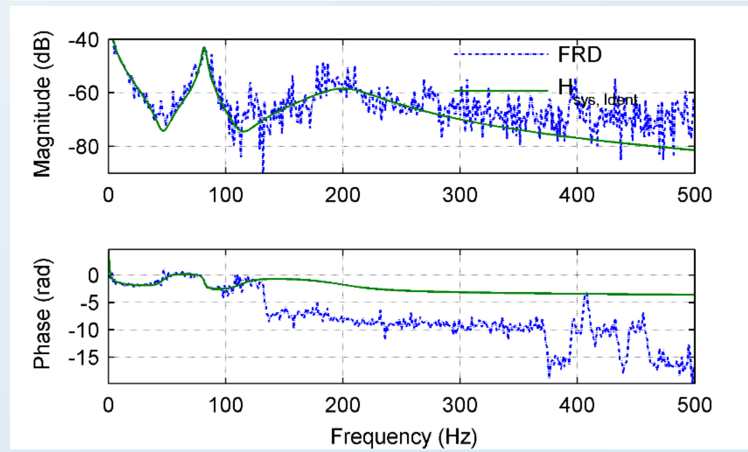
Simulation





Real system





chirp

PRBS

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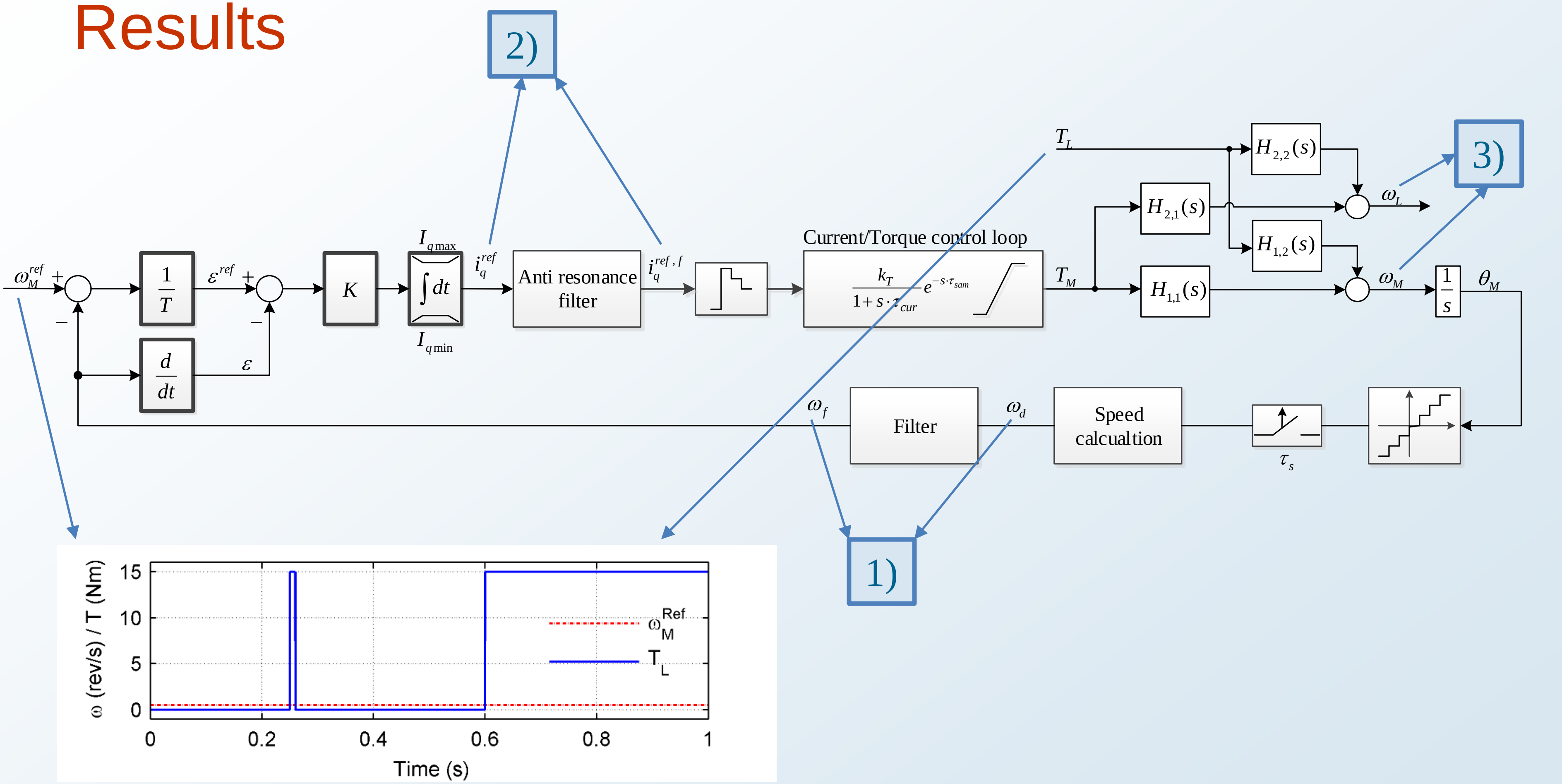
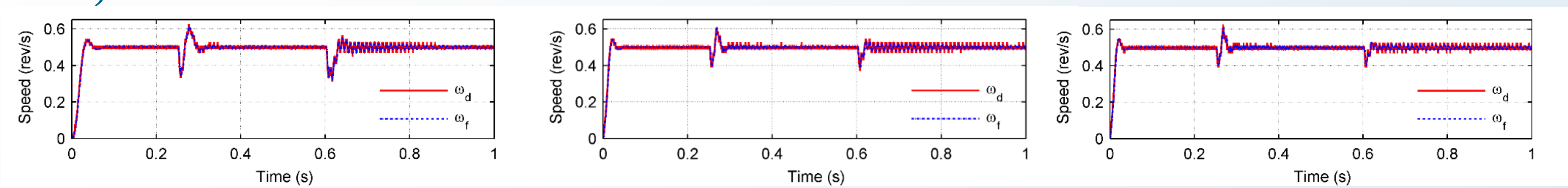
Results

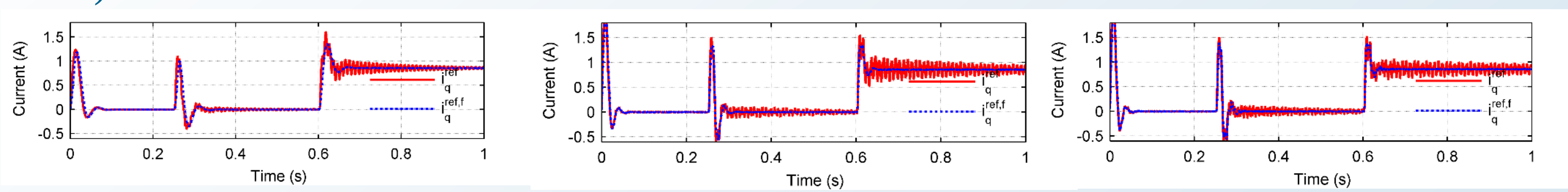

2)
3)
1)
ω_M^{ref}
$\frac{1}{T}$
ε^{ref}
$\frac{d}{dt}$
ε
K
$I_{q\max}$
$\int dt$
$I_{q\min}$
i_q^{ref}
Anti resonance filter
$i_q^{ref,f}$
Current/Torque control loop
$\frac{k_T}{1+s\cdot\tau_{cur}}e^{-s\cdot\tau_{sam}}$
T_L
T_M
$H_{2,2}(s)$
$H_{2,1}(s)$
$H_{1,2}(s)$
$H_{1,1}(s)$
ω_L
ω_M
$\frac{1}{s}$
θ_M
ω_f
Filter
ω_d
Speed calcualtion
τ_s
ω (rev/s) / T (Nm)
Time (s)
ω_M^{Ref}
T_L

Results

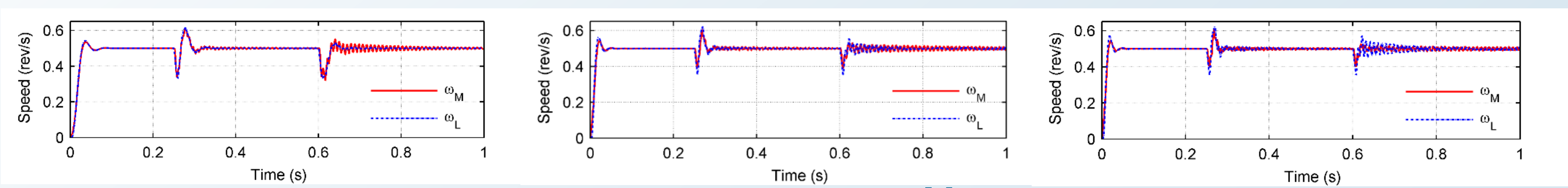
1)



2)



3)



Fwidth=175

Q=const=3.33 Fwidth i, i={32, 75, 105}

Tgr(0)=const

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Conclusions

1. Proper synthesis of speed controller requires:
   1. Identification of mechanical resonance frequencies.
   2. Tune digital filter to attenuate mechanical resonance frequencies in command signal.
   3. Calculate digital filter delay.
   4. Tune speed controller according to total system delay.
2. The tests show that increasing the filter delay negatively affects the dynamics of the control system. The delay time should be short. However reducing the time delay causes less attenuation of mechanical resonance band. As a result filter with a very low delay is to narrow.

Thank you for attention

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